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THE PLASTIC GROWTH OF A CAVITY NUCLEATED AT A SHEAR BAND

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Abstract—In this paper, the dynamic expansion of a spherical cavity nucleated at a shear band is theoretically investigated. To this end, the voided solid is represented in the vicinity of the band by a model consisting of a rigid-plastic thick hollow sphere which experiences an expansion characterized by a radial velocity field with spherical symmetry plus an additional pure shear distortional field representing localized shear deformation within a thickness $2h$. The selected shear strain rate field enables us to cope with localized deformation within a band of arbitrary thickness, oriented parallel to the direction of the imposed macroscopic shear strain. Expressions for the macroscopic deviatoric and volumetric stresses required to deform the material with voids located at shear bands were then computed. Furthermore, the dynamic void growth equation was also derived. The formulation was applied to the dynamic fracture of a thermal softening material in discrete steps, and the influence of the model parameters on the fracture strength of the material was assessed.

1. INTRODUCTION

Ductile fracture is a process caused by the nucleation and growth of microvoids, common nucleation sites being hard second phase particles or inclusions. Then, microvoids grow in a relatively stable manner until coalescence between neighbouring cavities takes place, giving rise to the formation of a fracture plane. Coalescence between voids may occur in two different manners, namely, by direct impingement or by strain localization (Curran *et al.*, 1987). Direct impingement is related to stable void growth involving large porosity values at the onset of fracture. However, strain localization within shear bands connecting voids enhances void nucleation along such bands, thus giving rise to a fracture plane at relatively low global porosity values.

Both analytical and numerical investigations on the behaviour of voided solids under different triaxiality conditions have been reported by several authors [see for instance, Rice and Tracey (1969), Duva and Hutchinson (1984), Ponte Castañeda and Willis (1988), Tvergaard (1981, 1982) and Becker *et al.* (1989)]. On the basis of an analytical approach, Gurson (1977) has been able to propose an approximate macroscopic yield function for voids containing solids, further studied by Tvergaard (1981, 1982). Moreover, the influence of void nucleation in the overall stress–strain response under simple tension and shear has been studied by Fleck *et al.* (1989).

The localization of strains in voided solids is a phenomenon which may be observed at low as well as at high loading rates. Shear bands appearing at very high loading rates are commonly referred to as adiabatic shear bands (Rogers, 1979). Strain localization is in this case enhanced by the net softening effect associated with the internal heat generation due to plastic deformation within the bands. Then, these bands may act as favoured nucleation sites originating a fracture plane along them (Rice, 1976; Rogers, 1979). So, an adequate knowledge of the dynamic growth of voids nucleated at shear bands is of crucial importance in the determination of the development of a fracture surface.

In this paper, we derive equations for the macroscopic response of a solid with voids which have nucleated at pre-existing shear bands. The simultaneous action of volumetric expansion and simple shear is considered, and the only effects contributing to the macroscopic deformation of the solid are supposed to be the shear deformation of the bands and the volumetric expansion of the voids contained in them. In the neighbourhood of the shear bands, the voided material is represented in an average sense by a rigid-plastic thick hollow sphere subjected to volumetric and deviatoric strain rate components, using an analogous approach to that presented by Gurson (1977). In Gurson's model, the deviatoric strain rate was assumed to be uniform over the thick hollow sphere. In this work, it is supposed that shear deformation is localized within a shear band of thickness $2h$. For simplicity, a non-conducting material is considered, in order to approximate to a situation of extremely high loading rates, where the time scale is small enough to prevent an important heat transfer in the deforming material. The dynamic void growth equation under these conditions is then determined, and an application to a thermal softening material in discrete steps is performed.

2. EQUATIONS FOR THE MACROSCOPIC STRESSES

2.1. *Velocity and strain rate fields*

In this section, we develop the formulation to be subsequently employed. The voided material is represented in the vicinity of the bands by a rigid-plastic thick hollow sphere which experiences a volumetric expansion defined by a radial velocity field with spherical symmetry, plus a pure distortion characterized by a uniform simple shear strain rate velocity field. The distortion is supposed to be concentrated within a pre-existing band of thickness $2h$, and symmetrically located with respect to a diametral plane of the sphere (see Fig. 1). It is expected that the nucleated cavities will have an initial radius a_0 less than the initial band semi-thickness h_0. However, as a result of the plastic void growth, the void radius a may eventually become larger than the band semi-thickness h. Thus, as shown in Fig. 1 both the cases $h \geqslant a$ and $h < a$ should be considered. Since a situation of simple shear is considered, the shear band is assumed to be oriented at a 45° angle with respect to the principal non-vanishing directions of deviatoric deformation. The initial inner radius a_0 of the sphere is supposed to correspond to the initial radius of the actual cavities, that is, at the onset of void growth. The initial outer radius b_0 of the sphere, also termed as the radius of influence of the cavity, is chosen in the case of a homogeneous distribution of voids in such a manner that $\xi_0 = (a_0/b_0)^3$, where ξ_0 is the initial porosity of the material (Carroll and Holt, 1972; Johnson, 1981; Cortés, 1992a). Thus, in the above case b_0 is about one half of the distance between neighbouring voids. In the present case, where the voids have nucleated along a sheet of material, b_0 will also be chosen as one half of the distance between neighbouring voids, measured along the bands. Thus, in our case ξ_0 should be interpreted as the local porosity value in the vicinity of the shear bands.

The radial velocity field is analogous to that defined by Carroll and Holt (1972), and is given by:

$$v_r = \dot{\varepsilon}_v b^3/3r^2, \tag{1}$$

where v_r is the radial velocity, $\dot{\varepsilon}_v$ is the volumetric expansion rate (defined as $\dot{V}/V$, V being

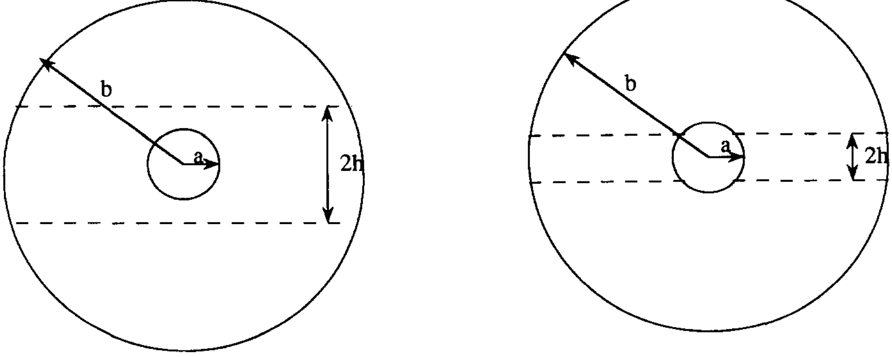

Fig. 1. Graphical representation of material modelling near a void nucleated at a shear band. a is the void radius, b is the radius of influence of the void and $2h$ is the band thickness: (a) for $h \geqslant a$; (b) for $h < a$.

the volume of the sphere including the void contained in it), b is the current outer radius of the sphere and r is the radial coordinate.

We consider now the deviatoric strain rate field which is only non-vanishing within a pre-existing band of thickness $2h$. This band is supposed to extend symmetrically about the diametral plane of the hollow sphere being parallel to the direction of the macroscopically imposed shear strain. Such a plane lies at a 45° inclination with respect to the principal axes of deviatoric deformation. We shall now establish the relationship between the microscopic components of the strain rate tensor and the macroscopic components of the strain rate tensor, macroscopic meaning the global behaviour of the material contained within the radius of influence of the voids nucleated at the bands. Thus, in this sub-section the word macroscopic will be employed in a restricted sense, since only a volume of material around the shear bands is being actually considered.

According to the definition of the macroscopic strain rate (Gurson, 1977), we can write:

$$\dot{e}_i = \frac{1}{V}\int_v \dot{e}_i^* \, \mathrm{d}V, \tag{2}$$

for $i = 1, 2, 3$, where the $\dot{e}_i^*$ denote the local deviatoric strain rate components and the $\dot{e}_i$ are the macroscopically imposed deviatoric strain rate components, both expressed in principal axes of deviatoric deformation. In the above equation V stands for the total macroscopic volume, including the void volume, which in our case corresponds to the volume of a sphere of radius b. In our particular problem, we have that $\dot{e}_i^*$ equals $\dot{e}_{hi}$ ($i = 1$, 2, 3) within the bands and vanishes outside the shear deforming zone. If the shear strain rate field is uniform within the deforming zone, then it is clear from eqn (2) that the deviatoric strain rate components $\dot{e}_{hi}$ ($i = 1, 2, 3$) within a band of thickness $2h$ will be given by:

$$\dot{e}_{hi} = \frac{V}{V_h}\dot{e}_i, \tag{3}$$

where V is the reference volume and V_h corresponds to the volume of the shear deformation zone, both including the void volume contained in them. In what follows, the ratio V/V_h will be designated by greek letter φ. Moreover, in our case the volume V_h can be readily computed as the remaining volume of a sphere after being cut along two parallel planes separated by a distance $2h$ and symmetrically located with respect to a diametral plane. Also, since $V = 4\pi b^3/3$, then parameter φ ($= V/V_h$) can be evaluated as:

$$\varphi = \frac{2}{3(h/b) - (h/b)^3}. \tag{4}$$

From all the above, we can write the deviatoric velocity field within the bands as:

$$v_i = \varphi \dot{e}_i x_i, \tag{5}$$

where the v_i are the Cartesian components of the velocity vector, the $\dot{e}_i$ are the components of the macroscopic deviatoric strain rate tensor and the x_i are the Cartesian coordinates. We remark that in this sub-section by macroscopic we mean the global behaviour of a volume of material around the shear bands. The repeated index in the right-hand side of eqn (5) does not involve summation. Therefore, the local deviatoric strain rate within the band will be $\varphi\dot{e}$ ($\dot{e}$ being defined as $(2\dot{e}_i\dot{e}_i/3)^{1/2}$), and it will be null outside the band.

Gurson (1977) has found an approximate expression, to the first order of approximation in the radial component $\dot{e}_r$ of the deviatoric strain rate field, for the local strain rate in a thick hollow sphere subjected to a macroscopic (and homogeneous) deviatoric strain rate $\dot{e}$ and to a volumetric strain rate $\dot{\varepsilon}_V$ as:

$$\dot{\varepsilon} = \sqrt{\dot{e}^2 + \frac{4\dot{\varepsilon}_v^2}{9\lambda^2}}, \tag{6}$$

where $\lambda = (r/b)^3$, r being the radial coordinate. It can be easily verified that in the conditions of the analysis and for any given radius within a shear band of an arbitrary thickness, the average strain rate value is given to the first order of approximation in $\dot{e}_r$ by an equation similar to eqn (6), namely:

$$\dot{\varepsilon} = \sqrt{\varphi^2\dot{e}^2 + \frac{4\dot{\varepsilon}_v^2}{9\lambda^2}}. \tag{7}$$

This latter equation will be used hereinafter to estimate the local strain rates in a shear band. Naturally, outside the bands the strain rate $\dot{\varepsilon}$ equals $2\dot{\varepsilon}_V/3\lambda$.

To avoid unnecessary complications, the $\dot{e}_i$ will be assumed to remain constant during the loading process. Moreover, since we will deal with a case of simple shear, we will impose that $\dot{e}_2 = -\dot{e}_1$ and $\dot{e}_3 = 0$. Thus, the shear strain rate $\dot{\gamma}$ equals $2\dot{e}_1$. Finally, it is worth noting that the plastic strain rates are supposed to be negligible outside the region of analysis, an assumption which is expected to be reasonable at the onset of fracture development along shear bands.

2.2. *Equations for the macroscopic volumetric and deviatoric stresses*

Gurson (1977) was able to show that the macroscopic deviatoric stress σ_d in equilibrium with first order homogeneous velocity fields in the strain rate components, such as that corresponding to eqns (1) and (5), was given by:

$$\sigma_d = \frac{1}{V}\int_v \sigma_e \frac{\partial\dot{\varepsilon}}{\partial\dot{e}}\,dV, \tag{8}$$

where $\dot{\varepsilon}$ is the local strain rate, $\dot{e}$ is the macroscopically applied deviatoric strain rate and σ_e is the local yield stress. On the other hand, the volumetric resistance to deformation is given by:

$$\sigma_V = \frac{1}{V}\int_v \sigma_e \frac{\partial\dot{\varepsilon}}{\partial\dot{\varepsilon}_V}\,dV. \tag{9}$$

Since we are dealing with first order homogeneous velocity fields in the components of the strain rate tensor, the above equations can be readily applied to our problem. We recall here that the plastic strain rates are supposed to be negligible outside the region of analysis, which implies that the only effects contributing the macroscopic deformation of the solid are the shear deformation at the bands and the volumetric expansion of the voids nucleated at them. Thus, it is easy to verify that even though we are analysing a limited region contained within the radius of influence of the voids, the so computed stresses are coincident with the actual macroscopic stresses required for plastically deforming the solid.

In this manner, eqn (8) yields the following expression for the macroscopic deviatoric stress:

$$\sigma_d = \frac{1}{V}\int_{Vh} \frac{\sigma_e\varphi^2\eta}{\sqrt{\varphi^2\eta^2 + 4/9\lambda^2}}\,dV, \tag{10}$$

where η is the ratio $\dot{e}/\dot{\varepsilon}_V$ and V_h refers to the volume of the deforming band.

For the macroscopic volumetric stress, eqn (9) yields:

$$\sigma_V = \frac{4}{9V}\int_{Vh} \frac{\sigma_e}{\lambda^2\sqrt{\varphi^2\eta^2+4/9\lambda^2}}\,dV + \frac{2}{3V}\int_{V-Vh} \frac{\sigma_e}{\lambda}\,dV. \tag{11}$$

Although it is expected that the nucleated cavities will have an initial radius a_0 less than the initial band semi-thickness h_0, the void radius a may eventually become larger than the band semi-thickness h as a result of the plastic void growth. Thus, as previously mentioned, both the cases $h \geqslant a$ and $h < a$ should be considered.

For simplicity, the yield stress σ_e will be subsequently assumed to depend on λ only. Strictly, this implies that σ_e should be viewed as an average value of the yield stress with respect to the angular coordinates. However, this drawback disappears if, for instance, σ_e is a constant over the domain of the integral, as will be the case in the model applications presented in Section 4. In this manner, for this simplified situation if $h \geqslant a$ (the void radius), eqn (10) becomes:

$$\sigma_d = \frac{3\varphi^2\eta}{2}\int_{\xi}^{\beta} \frac{\sigma_e\lambda\,d\lambda}{\sqrt{1+9\varphi^2\eta^2\lambda^2/4}} + \frac{3h\varphi^2\eta}{2b}\int_{\beta}^{1} \frac{\sigma_e\lambda^{2/3}\,d\lambda}{\sqrt{1+9\varphi^2\eta^2\lambda^2/4}}, \tag{12}$$

where ξ is material porosity, $\eta = \dot{e}/\dot{\varepsilon}_V$ as before and $\beta = (h/b)^3$. Also, eqn (11) gives:

$$\sigma_V = \frac{2}{3}\int_{\xi}^{\beta} \frac{\sigma_e\,d\lambda}{\lambda\sqrt{1+9\varphi^2\eta^2\lambda^2/4}} + \frac{2h}{3b}\int_{\beta}^{1} \frac{\sigma_e\,d\lambda}{\lambda^{4/3}\sqrt{1+9\varphi^2\eta^2\lambda^2/4}} + \frac{2}{3}\int_{\beta}^{1} \frac{\sigma_e(1-(\beta/\lambda)^{1/3})\,d\lambda}{\lambda}. \tag{13}$$

On the other hand, if $h < a$, the macroscopic deviatoric stress σ_d is given by:

$$\sigma_d = \frac{3h\varphi^2\eta}{2b}\int_{\xi}^{1} \frac{\sigma_e\lambda^{2/3}\,d\lambda}{\sqrt{1+9\varphi^2\eta^2\lambda^2/4}}, \tag{14}$$

whereas for the volumetric stress σ_V we obtain:

$$\sigma_V = \frac{2h}{3b}\int_{\xi}^{1} \frac{\sigma_e\,d\lambda}{\lambda^{4/3}\sqrt{1+9\varphi^2\eta^2\lambda^2/4}} + \frac{2}{3}\int_{\xi}^{1} \frac{\sigma_e(1-(\beta/\lambda)^{1/3})\,d\lambda}{\lambda}. \tag{15}$$

These latter equations implicitly define the macroscopic response of the solid with voids located at shear bands.

3. DYNAMIC VOID GROWTH EQUATION

We shall now derive the dynamic void growth equation for cavities nucleated at shear bands. The general expression for such an equation for a combined state of hydrostatic and deviatoric stresses is given by (Cortés, 1992b):

$$\sigma - \sigma_V = \sigma_I, \tag{16}$$

where σ is the applied macroscopic volumetric stress at the outer boundary of the sphere, σ_V is the plastic volumetric resistance to deformation of the material and σ_I is the inertia term. σ_V can be computed by use of eqns (13) or (15), while an explicit expression for σ_I has been previously derived as (Cortés, 1992b):

$$\sigma_I = \sigma_{IV} + \sigma_{ID}, \tag{17}$$

where

$$\sigma_{\rm IV} = \frac{\rho}{V}\int_V a_{\rm r}\frac{\partial v_{\rm r}}{\partial \dot{\varepsilon}_{\rm V}}\,{\rm d}V \tag{18}$$

and

$$\sigma_{\rm ID} = \frac{\rho}{V}\int_V a_{\rm R}\frac{\partial v_{\rm r}}{\partial \dot{\varepsilon}_{\rm V}}\,{\rm d}V. \tag{19}$$

In the above expressions $v_{\rm r}$ is the radial velocity field corresponding to the volumetric expansion of the sphere and $a_{\rm r}$ is its corresponding acceleration field. Moreover, $a_{\rm R}$ is the radial component of the acceleration associated with the deviatoric strain rate field, V is volume and ρ is mass density. The term $\sigma_{\rm IV}$ corresponds to the stresses required to overcome the inertia forces associated with the purely volumetric expansion, whereas the term $\sigma_{\rm ID}$ corresponds to the stress required to overcome the inertia effects associated with the interaction between the volumetric expansion and the deviatoric velocity fields. On integrating eqn (18) we obtain:

$$\sigma_{\rm IV} = \frac{\rho a_0^2}{3(\alpha_0-1)^{2/3}}\,Q(\ddot{\alpha},\dot{\alpha},\alpha), \tag{20}$$

where

$$Q(\ddot{\alpha},\dot{\alpha},\alpha) = \ddot{\alpha}[(\alpha-1)^{-1/3}-\alpha^{-1/3}]-(\dot{\alpha}^2/6)[(\alpha-1)^{-4/3}-\alpha^{-4/3}]. \tag{21}$$

In the above equation α represents the distention factor, which is related to material porosity by the expression $\alpha = 1/(1-\xi)$.

On the other hand, the integration of eqn (19) yields that for $h \geqslant a$, $\sigma_{\rm ID}$ is given by

$$\sigma_{\rm ID} = \frac{\rho\varphi^2\dot{e}^2h^2}{4}(1-(\xi/\beta)^{2/3})+\frac{3\rho\varphi^2\dot{e}^2ha_0}{8}(1-\beta^{1/3})[\alpha/(\alpha_0-1)]^{1/3}\left(1+\frac{\beta^{1/3}}{3}\right), \tag{22}$$

whereas for $h < a$, the corresponding expression is

$$\sigma_{\rm ID} = \frac{3\rho\varphi^2\dot{e}^2ha_0}{8}(1-\xi^{1/3})[\alpha/(\alpha_0-1)]^{1/3}\left(1+\frac{\beta^{2/3}}{3\xi^{1/3}}\right). \tag{23}$$

We will assume that, due to the local history of extremely high strain rates previously experienced by the material in the zone of localized shear deformation, such material exhibits local properties being highly different from those of the remaining material, due to local heating for instance. Moreover, since we are dealing with an incompressible non-conducting material, we can also assume for simplicity that the subsequent void growth will preserve the volume of such material with highly particular mechanical properties. Thus, the volume of such a zone of localized deformation will be assumed to remain constant throughout the subsequent cavity expansion process. To ensure this for $h \geqslant a$, it is required that the band semi-thickness h evolve as a function of the given local porosity value ξ as:

$$\frac{h}{b} = 2\cos\phi, \tag{24}$$

where

$$\phi = \tfrac{1}{3}\cos^{-1}\left(\frac{1-\xi}{1-\xi_0}\left(1+\frac{\beta_0}{2}-\frac{3\beta_0^{1/3}}{2}\right)-1\right), \tag{25}$$

where $\beta_0 = (h_0/b_0)^3$, and h_0, b_0 and ξ_0 being the values h, b and ξ, respectively, at the onset of void growth.

On the other hand, if $h < a$, the value of the band semi-thickness h ensuring volume constancy of the shear deforming zone will be given by the relation:

$$\frac{h}{b} = \frac{(1-\xi)}{(1-\xi_0)}\frac{1-\xi_0^{2/3}}{1-\xi^{2/3}}\frac{h_0}{b_0}. \tag{26}$$

We remark that it is expected that at the onset of void growth the initial void radius will be smaller than the band semi-thickness. Then, in this latter equation, h_0, b_0 and ξ_0 should represent the values of h, b and ξ corresponding to the instant when the condition $h < a$ was first accomplished.

From the above equations, eqn (16) can be numerically integrated for any given history of the externally applied volumetric stress pulse $\sigma(t)$, t being time.

4. APPLICATION TO THERMAL SOFTENING MATERIALS

4.1. *Constitutive equations for a thermal softening effect in discrete steps*

For illustrative purposes, and to emphasise the large quantitative differences in the mechanical response of the material within and outside the bands, we will suppose that the thermal softening effect of the material can be schematically represented by the expression:

$$\sigma_e = \sigma_0 H(T_0 - T) + \sigma_0^* H(T - T_0), \tag{27}$$

where T_0 is a critical temperature and $\sigma_0^* \leqslant \sigma_0$. H denotes the Heaviside step function, defined as $H(x) = 0$ if $x < 0$ and $H(x) = 1$ if $x \geqslant 0$. We will assume that at the onset of nucleation the temperature at the shear bands exceeds T_0, while the temperature outside the bands is much lower than T_0, in such a manner that the void growth does not affect the local mechanical response of the solid, that is, $\sigma_e = \sigma_0^*$ at the shear bands and $\sigma_e = \sigma_0$ outside the bands. In this simplified form, the net thermal softening of the material within the shear bands was taken into account. Under these assumptions, explicit expressions for the constitutive response of the material can be derived.

On integrating eqn (12), we obtain for $h \geqslant a$ that:

$$\sigma_d = \frac{2\sigma_0^*}{3\eta}\left(\sqrt{1+9\eta^2\varphi^2\beta^2/4}-\sqrt{1+9\eta^2\varphi^2\xi^2/4}\right)+\sigma_0^*\frac{h}{b}(\varphi/9\eta^2)^{1/3}S_0(\beta), \tag{28}$$

where $\beta = (h/b)^3$ as before. Function $S_0(x)$ is defined as:

$$S_0(x) = \tfrac{3}{2}\left((3\varphi\eta/2+\sqrt{1+9\eta^2\varphi^2/4})^{2/3}-(3\varphi\eta x/2+\sqrt{1+9\eta^2\varphi^2x^2/4})^{2/3}\right)$$
$$+\sum_{1}^{\infty} a_n\left((3\varphi\eta/2+\sqrt{1+9\eta^2\varphi^2/4})^{2/3-2n}-(3\varphi\eta x/2+\sqrt{1+9\eta^2\varphi^2x^2/4})^{2/3-2n}\right), \tag{29}$$

where the a_n are defined by:

$$a_n = \frac{(-1)^n(2/3)(-1/3)(-4/3)\dots(5/3-n)}{n!(2/3-2n)}. \tag{30}$$

Also, for $h < a$ the integration of eqn (14) yields:

$$\sigma_d = \sigma_0^* \frac{h}{b}(\varphi/9\eta^2)^{1/3} S_0(\xi), \tag{31}$$

where the function S_0 is again defined by eqn (29).

On the other hand, it follows from eqn (13) that for $h \geqslant a$ the volumetric stress will be given by:

$$\sigma_v = 2\sigma_0^*/3 \ln\left(\frac{\beta(1+\sqrt{1+9\varphi^2\eta^2\xi^2/4})}{\xi(1+\sqrt{1+9\varphi^2\eta^2\beta^2/4})}\right) + 2\sigma_0^*/3\,(h/b)(3\varphi\eta/2)^{1/3}G(\beta) + 2\sigma_0/3\,(\ln(1/\beta) - 3(1-\beta^{1/3})), \tag{32}$$

function $G(x)$ being defined as:

$$G(x) = 3\left(\frac{\sqrt{1+9\varphi^2\eta^2x^2/4}}{(3\varphi\eta x/2)^{1/3}} - \frac{\sqrt{1+9\varphi^2\eta^2/4}}{(3\varphi\eta/2)^{1/3}}\right) + 2^{1/3}S_0(x), \tag{33}$$

where $S_0(x)$ is again defined by eqn (29). If $h < a$, eqn (15) gives the following expression for the volumetric stress:

$$\sigma_v = 2\sigma_0/3(\ln(1/\xi) - 3\beta^{1/3}(\xi^{-1/3}-1)) + 2\sigma_0^*/3(h/b)(3\varphi\eta/2)^{1/3}G(\xi). \tag{34}$$

The above equations give, under the assumptions made, namely, negligible plastic strains outside the radius of influence of the cavities, the macroscopic response of a material obeying eqn (27).

4.2. *Numerical analysis of void growth*

For a material obeying eqn (27), the dynamic void growth equation was studied through the numerical integration of eqn (16). A constant volumetric stress rate of $\dot{\sigma} = 1$ GPa μs^{-1}, a deviatoric strain rate $\dot{e} = 10^3$ s^{-1}, an initial void radius of $a_0 = 10^{-6}$ m and an initial local porosity at the vicinity of the bands of $\xi_0 = 10^{-3}$ were considered. This porosity value corresponds to the case when voids have nucleated along shear bands, the distance between the centres of neighbouring cavities being 20 times its radius. Material properties of $\rho = 7850$ kg m^{-3} and $\sigma_0 = 700$ MPa were selected, which may correspond to steel. An explicit time integration scheme was used for the solution of eqn (16), the time step being 10^{-9} s. Since for the conditions of the analysis the term σ_{ID} given either by eqns (22) or (23), corresponding to the inertia effects due to the interaction of the radial velocity field and the deviatoric velocity field, was found to be extremely small, such a term was neglected and only the inertia term σ_{IV} [see eqn (20)] associated with the radial expansion velocity field was considered.

At each time step, and by use of eqns (16), (20) and (21), the second time derivative of α was evaluated, and the values of $\dot{\alpha}$, $\dot{\xi}$, $\dot{\varepsilon}_V$ and ξ were then updated. The analysis was continued until local porosity reached a value of about 0.3. This corresponds to a situation when the thickness of the ligament between neighbouring cavities [being equal to $2(b-a)$] equals the void radius a, a situation which is expected to be near the formation of a fracture plane by void coalescence along the shear band. The fracture strength was defined as the value of the stress corresponding to a porosity value of 0.3 but, as it will be seen later, such a definition of the fracture strength is in practice, for the cases studied, very insensitive to the porosity value associated to fracture. In the analysis, different values of the ratio h/b and of the softening parameter ζ, defined as $\zeta = \sigma_0^*/\sigma_0$, were considered.

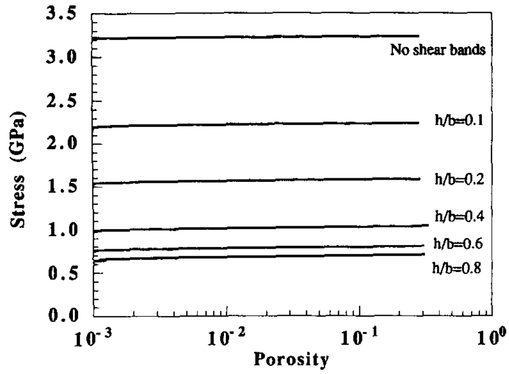

Fig. 2. Stress–porosity curves for a value of the softening parameter ζ ($=\sigma_0^*/\sigma_0$) of 0.2, for different initial values of the ratio h/b. The curve corresponding to the absence of shear bands is also shown.

Figure 2 shows the stress–porosity curves for a value of the softening parameter of $\zeta = 0.2$, and for the indicated initial values of the h/b ratio. The curve corresponding to the absence of shear bands has also been plotted. Initially, the stress increases with no porosity increase, a part of the curves which has been omitted. Then, when a threshold stress for void growth is exceeded, porosity starts to increase which corresponds to the part of the curve which has been actually represented. All the curves shown in Fig. 2 show a very large increase in porosity associated with a small increase in stress, in such a manner that the σ versus ln (ξ) curves are nearly horizontal for all the cases considered. As a consequence, it is virtually irrelevant to which porosity value the fracture strength is associated, since the arbitrariness of the selection of such a value will only introduce a very slight variation in the so defined fracture strength. For the case when no shear bands are present, we have obtained from the analysis that the fracture strength (that is, the stress corresponding to $\xi = 0.3$) is 3.24 GPa. If the voids have nucleated at a shear band with an initial value of $h/b = 0.1$, namely, the band semi-thickness h being equal to the void radius a, the fracture strength decreases to 2.24 GPa. If the initial value of h/b equals 0.2, the fracture strength decreases to 1.59 GPa, and if the initial value of h/b increases to 0.4, the fracture strength further decreases to 1.04 GPa. For initial values of the h/b ratio of 0.6 and 0.8, the corresponding fracture strengths are 0.81 and 0.71 GPa, respectively. Thus, we see the great influence of the initial value of the h/b ratio on the material fracture strength of the material.

In Fig. 3, the stress–porosity curves for an initial h/b ratio of 0.4 are depicted for the indicated values of the softening parameter ζ ($=\sigma_0^*/\sigma_0$). Again, only the part of the curves when the threshold stress for the void growth has been surpassed has been represented. It is also observed that the slopes of the σ versus ln (ξ) curves are very small, as in Fig. 2, making it irrelevant which porosity value is selected for the definition of the fracture strength. For a softening parameter of $\zeta = 0.8$, the fracture strength is 2.59 GPa, which decreases to 2.12 GPa for $\zeta = 0.6$. If ζ decreases to 0.4, the fracture strength decreases to

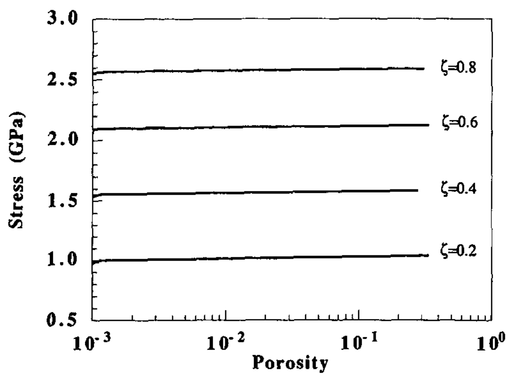

Fig. 3. Stress–porosity curves for an initial value of the ratio h/b equal to 0.4, corresponding to different values of the softening parameter ζ ($=\sigma_0^*/\sigma_0$).

1.58 GPa, and for $\zeta = 0.2$ the fracture strength is 1.04 GPa. Thus, the great influence that the softening parameter ζ has on the fracture strength of the material is evident.

5. DISCUSSION

In spite of the fact that a very simple constitutive model has been used to describe the material behaviour, it is expected that the main aspects influencing the fracture strength of a solid with voids located at shear bands have been portrayed. In fact, it has been estimated for a non-conducting linear thermal softening material, that the thermal softening by itself has a negligible influence on the fracture strength of a voided material under simple volumetric expansion, since the heat generated by plastic deformation excessively localizes near the surface of the voids (Cortés, 1992a). Thus, it is expected that for a linear thermal softening material, the heat generated during the short period of time required for the development of a fracture plane, with its consequent thermal softening effect, will only have a minor importance in the fracture process with respect to other much more important factors. Such factors are the previous deformation at the shear band at the onset of cavity nucleation, and the initial value of the h/a (or h/b) ratio, apart from the initial local porosity value. The first of such factors relates to the heat generation by plastic deformation and, thus, to the net thermal softening effect experienced by the material within the shear bands previous to void nucleation. From the numerical analysis performed, it follows that the fracture strength falls dramatically with the softening effect within the bands, that is, with the decrease of parameter ζ. The second of the above factors relates with the relative volume of softened material around the voids. Clearly, the volumetric expansion of the solid is easier as the relative volume of softened material (i.e. the h/b ratio) increases. From the numerical analysis performed, we also appreciate that the fracture strength decreases enormously as the initial value of the ratio h/b (or h/a) increases. The third parameter, namely, the local initial local porosity ξ_0 at the bands, has been defined in the text as $(a_0/b_0)^3$, where a_0 and b_0 are respectively the initial void radius and one half of the distance between the centres of neighbouring voids measured along the plane of the shear band. It is clear that as the distance between the cavities decreases, thus increasing the initial local porosity value, the fracture strength of the material will decrease. In fact, calculations performed for $\xi_0 = 10^{-2}$, and for an initial value of $h/b = 0.4$ and a softening parameter of $\zeta = 0.2$, yielded a fracture strength of 0.81 GPa, appreciably lower than the value of 1.04 GPa found for an initial local porosity value of $\xi_0 = 10^{-3}$. Thus, a decrease in the distance between neighbouring cavities will also raise an important decrease in the fracture strength of the material.

6. CONCLUSIONS

In this paper, explicit expressions for the mechanical response in the vicinity of shear bands, of a rigid-plastic material with voids nucleated at the shear bands have been derived for a situation of mixed volumetric expansion and simple shear. Under the assumption that the plastic strains are negligible farther than the radius of influence of the voids, the derived expressions correspond to the macroscopic constitutive equations of the solid. The dynamic void growth equation has been also derived. The above equations have been applied to the case of a thermal softening material in discrete steps, which illustrates the situation when the material within the band has become greatly softened with respect to the material outside the bands, due to localized plastic shear deformation, and the associated local heat generation. The influence of parameters such as the normalized band thickness h/b and the softening parameter ζ $(=\sigma_0^*/\sigma_0)$ upon the fracture strength of the material were assessed. It was concluded for the conditions of the analysis, that the fracture strength greatly decreases as the ratio h/b increases, and as the softening parameter ζ decreases.

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